known as the **characteristic equation**. If M is nonsingular, then the there are $2n$ eigenvalues and eigenvectors. The problem of finding the eigenvalues and eigenvectors of $P(\lambda)$ is known as the **quadratic eigenvalue problem (QEP).**

The underlying equation, which is often used in dynamic analysis of mechanical systems, is a homogenous linear second-order differential equation:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = 0. \tag{2.1.3}$$

Mechanical structures are usually modeled by the equations, which are typically obtained by finite element discretization of distributed parameter systems.

Using separation of variables and assuming a solution of the form $q(t) = \phi_0 e^{\lambda_0 t}$, the equation (2.1.3) leads us to the eigenvalue-eigenvector problem:

$$P(\lambda_0)\phi_0 = 0.$$

In the case, when all of the eigenvalues of the quadratic pencil are distinct, the general solution to the above equation (2.1.3) is:

$$q(t) = \sum_{k=1}^{2n} a_i \phi_i e^{\lambda_i t}.$$

More generally, when λ_0 is an eigenvalues of algebraic multiplicity p, function

$$q(t) = \left(\frac{t^k}{k!}\phi_0 + ... + \frac{t}{1!}\phi_{k-1} + \phi_p \right) e^{\lambda_0 t}$$

is a solution of the differential equation if the set of vectors $\phi_0, ..., \phi_p$, with $\phi_0 \neq 0$, satisfies the relation

$$\sum_{p=0}^{j} \frac{1}{p!} L^{(p)}(\lambda_0)\phi_{j-p} = 0, \;\; j = 1, ..., p.$$

Here $L^{(p)}$ is the pth derivative of the polynomial. Such set of vectors $\{\phi_1, ..., \phi_p\}$ is called a *Jordan chain* of length $p+1$ associated with eigenvalue λ_0. The Jordan